Irreversibility and metastability in spin-glasses. II. Heisenberg model

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We numerically compute the various history-dependent magnetizations for Heisenberg spin-glasses with and without anisotropy. The exchange interactions are of short range and have a Gaussian probability distribution. Our approach closely follows that of paper I. In the absence of anisotropy, a Heisenberg spin-glass is found to have no irreversibility. The field-cooled and zero-field-cooled magnetizations are macroscopically equivalent and magnetic hysteresis is absent. This macroscopic reversibility is a consequence of the accessibility of the rotationally degenerate field-cooled state and does not correspond to microscopic reversibility. Both Dzyaloshinsky-Moriya (DM) and uniaxial anisotropy introduce macroscopic irreversibility. In the latter case the hysteresis loops are like those which we found for Ising spins. In the former case, in some situations, we find displaced loops, which look similar to those seen in Mn-containing spin-glasses. To get qualitative agreement with experiment, we must also impose a tendency towards ferromagnetism. This ferromagnetic tendency (which corresponds to a displaced Gaussian exchange distribution with $J_0>0$) is essential in order to maintain rigid rotation of the spins in response to field rotations. This rigidity is a fundamental assumption in other approaches which explain analytically why DM anisotropy leads to displaced hysteresis loops. Finally we study the coexistent (longitudinal) ferromagnetic—(transverse) spin-glass phase proposed by Gabay and Toulouse. The behavior of the coexistent spin-glass is very similar to that of typical spin-glasses in very large applied fields. We see no indication for reentrant behavior, as is often observed experimentally, in our temperature-dependent moderate-field magnetizations. Furthermore, a calculation of the zero-field (J_0,T) phase diagram for isotropic systems shows no evidence for reentrant behavior. We cannot rule out the possibility that a reentrant transition exists only in some narrow range of low magnetic fields.

I. INTRODUCTION

In this paper the various history-dependent magnetizations for Heisenberg spin-glasses are computed. We discuss the role that microscopic anisotropy plays in irreversible phenomena and investigate the recently proposed[1,2] coexistent spin-glass—ferromagnetic phase. The approach used here is identical to that of the preceding paper[3] (called I). Some of this work was summarized in an earlier Letter.[4]

As discussed in I, it is assumed that on an "intermediate" time scale, a spin-glass will follow a given minimum of the free energy as the surface evolves with field H or temperature T. Magnetic hysteresis and temperature-dependent irreversibility arise because of the disappearance of the minimum upon changing H or T; this, in turn, causes the system to "hop" to a new nearby metastable state and leads to irreversibility. "Intermediate" times correspond to those which are sufficiently long to allow the spin-glass to find the nearest minimum, after H or T has changed, but sufficiently short so that no tunneling or thermal activation processes will take the system to another metastable state.

In our calculations we are forced, by default, to look at the simplest mean-field model for the free-energy functional $F[\{\vec{m}_i\}]$. Here $\vec{m}_i$ is the thermal average of the spin at the ith site. The corrections to mean-field theory, deriving from the "reaction terms," lead to unphysical results.[5] These problems probably arise from errors in standard calculations of the reaction term, which errors are, in turn, due to finite-size effects. Since, in the Ising model, mean-field theory has led to useful insights into the nature of the free-energy surface, it is important to apply it also to the Heisenberg case. It should be noted that $(T=0)$ ground states derived in mean-field theory also satisfy the condition for metastability used in Monte Carlo simulations.[6] At zero temperature the neglect of the reaction terms is justified and all of our $T=0$ results should be qualitatively, if not quantitatively, the same as in Monte Carlo simulations. What differs in the two approaches is the algorithm which determines how the system "flows" over the $T=0$ free-energy surface.

In comparison with Ising systems, because of the expense, there is virtually no simulation work on history-dependent properties of Heisenberg spin-glasses. Walstedt and Walker[6] have pointed out that anisotropy plays a key role in the Heisenberg case. Without anisotropy they see no evidence for a cusp in the zero-field "equilibrium" magnetic susceptibility, which is a signature of the spin-glass phase. Two types of anisotropy have been studied, using primarily analytical methods. Fert and Levy have focused attention[7,8] on Dzyaloshinsky-Moriya[9] (DM) anisotropy; they and others[10–12] have constructed a macroscopic free energy based on this microscopic anisotropy. The assumption of rigid rotation of the spins is fundamental in their work. Our approach allows us to examine this assumption in some detail. Roberts and Bray[13] and Cragg and Sherrington[14] have studied uniaxial anisotropy using replica-symmetry-breaking techniques[15] applied to the infinite-range interaction model. This theoretical approach cannot lead to a direct calculation of the various history-dependent magnetizations. However, it does claim to specify under what circumstances irreversible behavior will occur. Uniaxial anisotropy has been experimentally[16] studied in a class of spin-glasses having hexagonal crystal structure. The work of Cragg and Sherrington makes contact with these interesting systems.

Although mean-field theory represents a clear oversimplification, our numerical approach yields new and previously unavailable information about the effects of microscopic anisotropy mechanisms on macroscopic measurements. We are able to calculate the field-cooled (FC) and zero-field-cooled (ZFC) magnetizations as functions of temperature and field, and determine how they are affected by DM and uniaxial anisotropy. Hysteresis loops are obtained in the presence of both types of anisotropy. The so-called "displaced" loops[17,18] are found only when the anisotropy is of the DM type. This result supports a previous claim made by Fert and Levy.[7] As in the Ising case (of paper I), we see sharp magnetization reversals in hysteresis loops, only when there is a net tendency towards ferromagnetism. In this previous paper we found that the experimentally observed behavior of the magnetic hysteresis in the strongly anisotropic *Au*Fe alloys (for a range of Fe concentrations) was similar to that obtained in our calculations based on the Ising model. In this paper we show how many features observed in *Cu*Mn hysteresis experiments can be explained using a Heisenberg Hamiltonian with weak DM anisotropy.

A very striking result of our calculations (which was first reported elsewhere[4]) is that in an *isotropic* Heisenberg spin-glass there is no macroscopic irreversibility. The FC and ZFC states are found to be the same and magnetic hysteresis is absent. This lack of measurable irreversibility holds despite the fact that changing H or T affects the free-energy surface for Heisenberg spins in the same way as was found for the Ising case: (i) Minima generally disappear upon heating, but never upon cooling and (ii) at sufficiently low temperatures changing the field by small amounts leads to the disappearance of a given minimum. Thus in the Heisenberg case, as in the Ising model, there is "minima hopping." The unique aspect of the isotropic Heisenberg spin-glass is that the system evidently hops between minima all of which correspond to (rotationally degenerate) field-cooled states. That is, because of the rotational degeneracy of the FC (and all other states) in the plane perpendicular to the field, the FC state in particular is extremely accessible. In the isotropic Heisenberg model there is microscopic but not macroscopic irreversibility. Upon completion of a hysteresis "loop" we find that the two values of the $\vec{m}_i$ for each H (corresponding to the two "legs" of the loop), are different, but that the total magnetization at a given H is independent of the history.

An important consequence of this result is that, on these intermediate time scales, there is no magnetic remanence. Because the present theory and the Monte Carlo approach are equivalent at $T=0$, it should be noted that our observations of vanishing remanence and magnetic hysteresis can be corroborated in Monte Carlo simulations. There is some preliminary simulation evidence[19] to support our results. A Heisenberg spin-glass with no anisotropy can essentially "follow" a changing magnetic field. As discussed in I, the inertial behavior that we associate with magnetic remanence occurs when the spin-glass gets trapped in a metastable state (of nonzero magnetization). Evidently this trapping can only happen when anisotropy is present. Once anisotropy is introduced, the FC and ZFC states become macroscopically distinct; the behavior of the temperature-dependent magnetizations is qualitatively the same as found in the Ising case, and closely resembles experimental results. Because of the similarity of the Ising and anisotropic Heisenberg models, we do not present new calculations for all the different spin-glass properties stuided in I, such as magnetic remanence or the field-dependent specific heat.

In this paper we also briefly discuss the xy model, which is a special case of the uniaxially anisotropic Heisenberg system. As might be expected, for fields applied in the x-y plane, the FC and ZFC magnetizations are inequivalent. However, the splitting between these two is somewhere between that of the isotropic Heisenberg and Ising models.

An outline of the paper is as follows: In Sec. II we present the model Hamiltonian and outline the theoretical framework for dealing with uniaxial and Dzyaloshinsky-Moriya (DM) anisotropy. The latter is readily handled within mean-field theory, whereas the former (for $S=1$) requires some matrix and numerical analysis. Section III contains a discussion of several phase diagrams (for the various anisotropy constants as functions of temperature). We also present results for the transverse and longitudinal order parameters as well as the respective transition temperatures. The field dependence of the transversely ordered state is treated in some detail. Section IV describes results for the temperature dependence of the FC and ZFC magnetizations when various types of anisotropy are present. The behavior of the x-y spin-glass model is also summarized. In Sec. V we discuss the results of a number of magnetic hysteresis calculations. Of particular interest is the presence of displaced loops observed, under the proper circumstances, when DM anisotropy is present. The response of isotropic and anisotropic spin-glasses to field rotations is considered in Sec. VI. In general, rigid rotations are not observed unless the field values are extremely large or a ferromagnetic tendency is present. Section VII deals with the Gabay-Toulouse[1] coexistent spin-glass ferromagnet. The temperature-dependent, moderate-field magnetizations and zero-field phase diagram are not found to be suggestive of reentrant (or disappearing) ferromagnetism, despite the fact that experimentally this phenomenon is claimed to exist. Finally, in Sec. VIII we list our conclusions. While our results are generally presented for one distribution of the exchange interaction, we have ascertained that all our numerical plots are fairly typical.

Unlike the preceding paper, we do not present many experimental results for comparison purposes. This is primarily because once anisotropy is introduced, the vector spin-glass properties are generally similar to those we saw in Ising systems. Thus qualitative comparison between our theory and experiment can be made by reference to paper I. The main exception is our discussion of displaced hysteresis loops in which we present several experimental results. It should be stressed that, as in I, it is not appropriate to make quantitative comparison with the data.

We deal with an oversimplified model Hamiltonian, treated within the mean-field approximation and solved for finite-size systems.

II. DESCRIPTION OF THE METHOD

We consider the Heisenberg Hamiltonian with nearest-neighbor interactions

$$\mathscr{H}=\sum_{i,j} J_{ij}\vec{S}_i\cdot\vec{S}_j-\sum_i D(S_i^z)^2-\sum_{i,j}\vec{D}'_{ij}\cdot(\vec{S}_i\times\vec{S}_j)-\sum_i \vec{S}_i\cdot\vec{H}\ . \qquad (2.1)$$

The first term represents the random exchange interactions between spins i and j, which are distributed according to a Gaussian probability distribution $P(J_{ij})$ of width $\bar{J}$ and center J_0. The second and third terms correspond to uniaxial and Dyzaloshinsky-Moriya anisotropy, respectively. Here $\vec{H}$ is the external magnetic field and the magnetic moment $g\mu_B\equiv 1$; all energies (J_0, D, H, etc.) are measured in units of $\bar{J}$.

It is also assumed that the coefficients $D'^{\mu}_{ij}=-D'^{\mu}_{ji}$ are distributed randomly. Here, and throughout, μ and ν are taken to be Cartesian coordinates. For convenience, we chose a two-δ-function distribution for the D'^{μ}_{ij}: $P[D'^{\mu}_{ij}]=\delta(D'^{\mu}_{ij}\pm D')$. The anisotropy constants D and D' were chosen as variable parameters, in our numerical studies. (Thus D was allowed both positive and negative signs.) The limit $D\rightarrow-\infty$ corresponds to an x-y model, which will be discussed in Sec. IV. Within the context of mean-field theory, the first and third terms in Eq. (2.1) can be combined, so that their total molecular field $\vec{H}_i$ is given in terms of a tensor $\overleftrightarrow{J}_{ij}$

$$H_i^{\mu}\equiv\beta\sum_{j,\nu} J_{ij}^{\mu\nu}m_j^{\nu}+\beta H\delta_{\mu z},\quad \beta^{-1}=k_BT\ . \qquad (2.2)$$

for $\vec{H}$ applied in the $\hat{z}$ direction. Here

$$\overleftrightarrow{J}_{ij}=\begin{vmatrix} J_{ij} & D'^{z}_{ij} & -D'^{y}_{ij} \\ -D'^{z}_{ij} & J_{ij} & D'^{x}_{ij} \\ D'^{y}_{ij} & -D'^{x}_{ij} & J_{ij}\end{vmatrix}\ . \qquad (2.3)$$

In the absence of uniaxial anisotropy, the condition that $\vec{m}_i$ be an extremum of the (mean-field) free energy is

$$\vec{m}_i=\vec{H}_i\frac{SB_s(|\vec{H}_i|)}{|\vec{H}_i|}\ , \qquad (2.4)$$

where B_s is the usual Brillouin function for spin S.

We considered uniaxial anisotropy only for the case $S=1$, since for general S, a fair amount of matrix algebra and numerical analysis is required. Thus in all calculations throughout this paper we treat only $S=1$. It should be noted that in previous work,[13,14] which studied uniaxial effects, it was assumed that the Heisenberg spins were classical. In this way matrix analysis was avoided. For definiteness we outline our procedure for the case when the only anisotropy present is uniaxial. When $S=1$, the mean-field Hamiltonian for the ith site can be written as a 3×3 matrix, using the generalized ($S=1$) Pauli matrices. It can be readily shown that the self-consistent equation for $\vec{m}_i$, in terms of $\vec{H}_i$ in Eq. (2.2) (with $\vec{D}'_{ij}=0$), is

$$m_i^{\mu}=-\sum_{k=1}^{3}\frac{-2DH_i^{\mu}-2E_kH_i^{\mu}}{-D^2+(H_i^{\mu})^2-4DE_k-3E_k^2}\,\frac{e^{-\beta E_k}}{\sum_{k'=1}^{3}e^{-\beta E_{k'}}} \qquad (2.5\text{a})$$

for $\mu=x$ and y and for $\mu=z$ the term $-2DH_i^{\mu}-2E_kH_i^{\mu}$ in Eq. (2.5a) is replaced by $-2E_kH_i^z$. Here E_k is obtained by solving for the three cubic roots of

$$E_k(-D^2+H_i^2)-2DE_k^2-E_k^3+D[H_i^2-(H_i^z)^2]=0\ , \qquad (2.5\text{b})$$

where $H_i^2\equiv\sum_{\mu}(H_i^{\mu})^2$. Equations (2.5) reduce to Eq. (2.4) in the limit $D=0$, as expected.

It should be apparent from Eq. (2.4) that at $T=0$ (when uniaxial anisotropy is absent) $\vec{m}_i$ is parallel to $\vec{H}_i$. This criterion for metastability coincides with that used in ground-state Monte Carlo simulations.[6] A convincing argument for its validity is given in Ref. 20. In the case of an xy model with infinite-range interactions our (ground-state) effective-field distribution is the same as that obtained by Palmer and Pond[21] using Monte Carlo simulations. It is important to note that when uniaxial anisotropy is present, it is not possible to define a total molecular field $\vec{H}_i$ along which $\vec{m}_i$ points. The way in which uniaxial anisotropy helps to orient the spins is fairly subtle, as can be seen from Eqs. (2.5).

As in I, we solved Eqs. (2.4) and (2.5) numerically using an iterative technique. Convergence is assumed when

$$\frac{\sum_i[(\vec{m}_i)_n-(\vec{m}_i)_{n-1}]^2}{\sum_i[(\vec{m}_i)_n]^2}\le 10^{-6}\ . \qquad (2.6)$$

Here n represents the nth iteration. Ling *et al.*[5] have shown that all solutions for the $\vec{m}_i$ which are obtained by iterative convergence are minima of the free energy. This result has been rigorously proven for the Heisenberg case when no updating procedure (see I) is used. It also holds when anisotropy is present. As discussed in I, it is not likely that updating Eq. (2.4) (which is necessary in order to expedite convergence) will lead to problems.

Because of the relative complexity of treating vector spins, we were forced to consider somewhat smaller systems than in the Ising model. $N\approx10^3$ for the DM and isotropic Heisenberg cases. The presence of uniaxial anisotropy requires the numerical solution of a cubic equation [Eq. (2.5b)] at each iteration. Therefore, in this case we considered $N\cong6^3$ spins. Our procedures for generating field-cooled states, zero-field-cooled states, and hysteresis loops were the same as those used in paper I. Convergence, however, typically required many more iterations than in the Ising case. We have verified that our results are unaffected when a more stringent convergence criterion is used in Eq. (2.6).

On occasion, field-cooling processes led to numerical difficulties, particularly, in large fields. At high T, the $\vec{m}_i$ were aligned parallel to the field direction (say $\hat{z}$), and as the system was cooled, there was often a delay before the $\hat{x},\hat{y}$ components were able to become nonzero. That is, the spin-glass tended to supercool into a longitudinally ordered state, so that $m_i^x = m_i^y = 0$. This was generally avoided by applying a very small perpendicular field in the $\hat{x}+\hat{y}+\hat{z}$ or $\hat{x}+\hat{y}$ direction of magnitude $10^{-4}\bar{J}$. We verified that our results were *quantitatively* insensitive to the value of this field, but it was essential that it be present, particularly in the case of uniaxial anisotropy. In all cases the states we found in this way had lower free energy than the supercooled state.

III. THE PHASE DIAGRAM AND TEMPERATURE-DEPENDENT ORDER PARAMETER

As in I, we take the spin-glass order parameter to be of the Edwards-Anderson type. However, since $\vec{m}_i$ is a vector, the order parameter [which involves $(m_i^\mu)^2$] may have up to three components. When $\vec{H} \neq 0$ or $D \neq 0$, the spin-glass is no longer isotropic so that there are two distinct terms called $Q_{\parallel}$ and $Q_{\perp}$, where the subscripts refer to the direction defined by $\vec{H}$ or the anisotropy axis (called z). (When $\vec{H}$ and D are both present and noncollinear, a three-component order parameter is needed.) Unless indicated otherwise, we will not treat this case in this paper. We define

$$Q_{\perp} \equiv \frac{1}{2N} \sum_i [(m_i^x)^2 + (m_i^y)^2] , \tag{3.1a}$$

$$Q_{\parallel} \equiv \frac{1}{N} \sum_i (m_i^z)^2 . \tag{3.1b}$$

There are thus, in principle, two transition temperatures T_c corresponding to the onset of longitudinal ($T_{c\parallel}$) and transverse order ($T_{c\perp}$). Throughout this paper each T_c is found by extrapolating our finite-N results to the thermodynamic limit. For $D > 0$ and/or $H \neq 0$ the higher transition corresponds to the ordering of $Q_{\parallel}$. For $D < 0$, $Q_{\perp}$ orders first. For $D \to \infty$, the spin-glass is Ising type, whereas for $D \to -\infty$, it is an x-y system.

There have been several calculations[13,14] of the phase diagram in the D-T plane for a classical Heisenberg spin-glass with infinite-range interactions (and with $\vec{D}'_{ij} = 0$). Our phase diagrams look qualitatively similar to those previously obtained, except that the maximum value of D, which corresponds to the onset of only longitudinal ordering at all temperatures, is different. We find $D^{\max} = 1.7$ and $D^{\min} = -0.20$, as compared with $+0.32$ and -0.20 for the infinite-range classical spin model.

In Fig. 1(a) is plotted the temperature dependence of the two order parameters for different values of the uniaxial

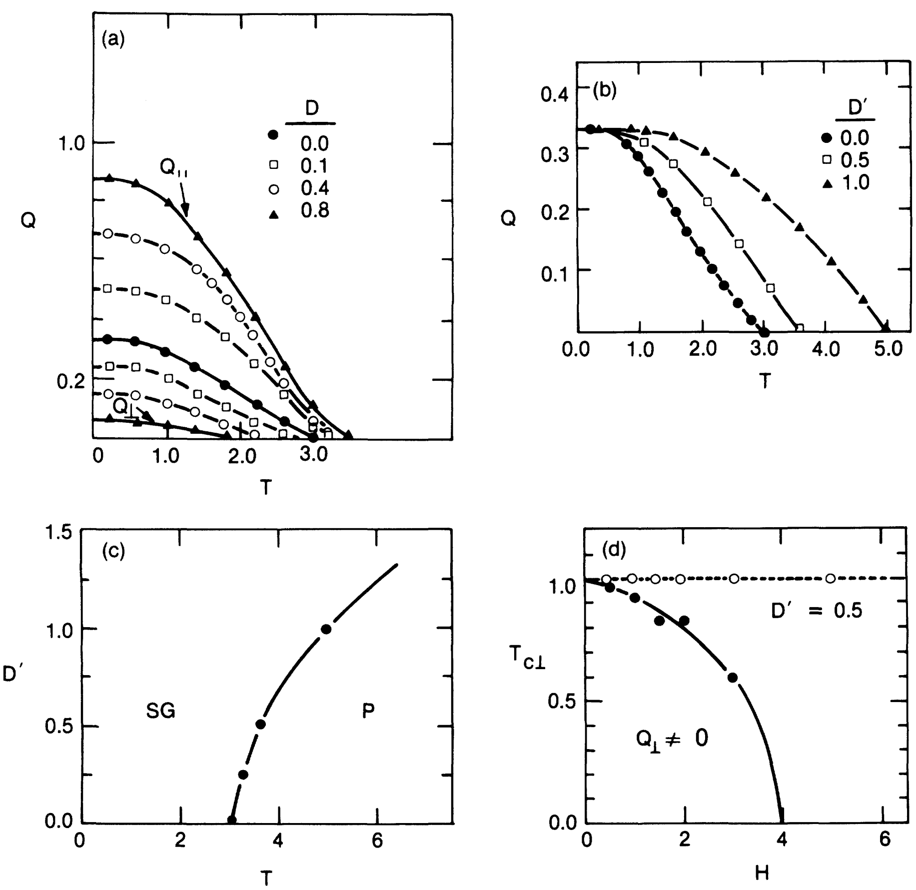


FIG. 1. (a) Temperature dependence of spin-glass order parameters ($Q_{\parallel}$ and $Q_{\perp}$) for different values of uniaxial anisotropy with $N = 6^3$. All energies are units of $\bar{J}$, and $S = 1$ in all calculations. (b) Temperature dependence of order parameter $Q_{\parallel} = Q_{\perp}$ for Dzyaloshinsky-Moriya anisotropy. (c) Phase diagram for spin-glass with pure DM anisotropy. (d) Field dependence of transverse ordering temperature for isotropic (●) and DM anisotropic Heisenberg system (○). $T_{c_{\perp}}$ is normalized to its value when $H = 0$. In (a)–(c), $H = 0$.

anisotropy $D \geq 0$ and $H=0$. No DM anisotropy is present. Here $N=6^3$ or 10^3. This figure shows clearly that $Q_{\parallel}$ orders first and that for sufficiently large D, the transverse ordering will be totally suppressed. The presence of a magnetic field or finite $J_0>0$ raises $Q_{\parallel}$ and suppresses $Q_{\perp}$. The various values of $T_{c\parallel}$ and $T_{c\perp}$ can be read off the figure. $T_{c\parallel}$ increases with D, whereas $T_{c\perp}$ is lowered. For $D=0$, $T_c \sim 3.1\bar{J}$.

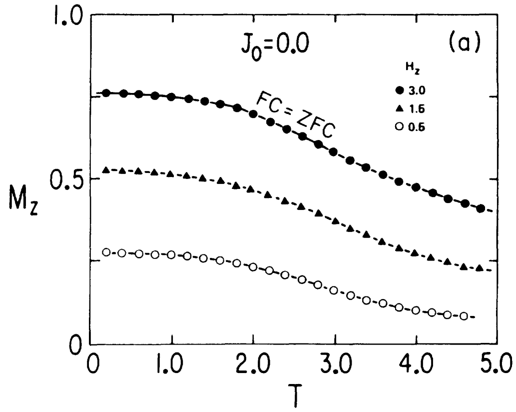
For the case of pure DM anisotropy there is clearly no anisotropy in Q, unless a magnetic field is present. In Fig. 1(b) is plotted $Q_{\parallel}=Q_{\perp}$ versus temperature for this case. We took $N=10^3$ and considered three values of the anisotropy constant D'. As may be seen, the larger D', the higher is the T_c. The anisotropy acts to strengthen the random interactions in the system. Some of the values of the anisotropy constants in Figs. 1(a) and 1(b) were chosen to be larger than is probably physical for purposes of illustration. We found that occasionally inconsistent results were obtained when $D'>0.5$. Extremely high values of DM anisotropy correspond to a highly "frustrated" system, particularly since this anisotropy has a peculiar vector nature. By constrast, we had no difficulties increasing the uniaxial anisotropy D to arbitrarily large negative or positive values.

The results in Fig. 1(b) can be used to construct a phase diagram for the DM case. In Fig. 1(c) is plotted this D'-T diagram. This should be contrasted with that obtained[13,14] for the uniaxial case. Here there are no intermediate phases corresponding to purely longitudinal or purely transverse ordering. We found that for small values of the DM anisotropy T_c increases slightly slower than linearly. Because of the oversimplifications of the model used here, it is not clear whether this result can be related to any experimental situation. Presumably in a laboratory spin-glass it is not possible to vary D' without affecting other parameters as well.

Finally, in Fig. 1(d) is plotted the field dependence of $T_{c\perp}$ for the isotropic Heisenberg case and for the case of pure DM anisotropy. As has been found in analytical calculations[1,2] based on the infinite-range model, $T_{c\perp} \sim T_c(1-aH^2)$ for small H. The coefficient of the quadratic term we find is 0.18 if H is measured at units of T_c; this agrees with analytical work[1,2] in which $a=0.23$. By $H=4.0$ there is no longitudinal spin-glass order. By contrast, when pure DM anisotropy is present, we see virtually no reduction of the perpendicular ordering temperature up to extremely large fields. Evidently because of the vector cross-product nature of the anisotropy, the system has difficulty suppressing entirely the ordering in the $\perp$ direction. It should be stressed, however, that the magnitude of $Q_{\perp}$ will be very much reduced relative to $Q_{\parallel}$ in large fields.

IV. RESULTS FOR THE TEMPERATURE-DEPENDENT MAGNETIZATION

In this section we present results for the FC and ZFC magnetizations as functions of temperature. We consider a range of values for the parameters J_0, D, and D' corresponding, respectively, to the center of the Gaussian distribution of the $\{J_{ij}\}$, the uniaxial, and DM anisotropy constants.



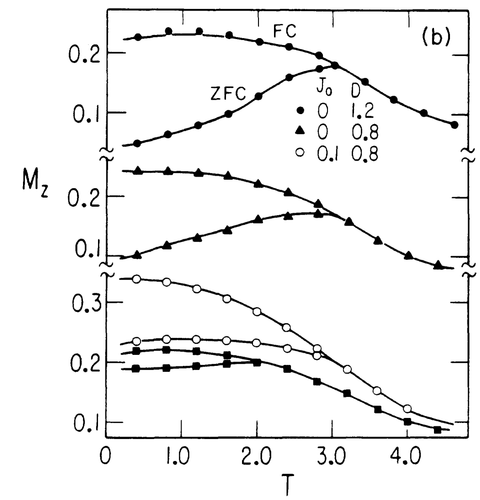


FIG. 2. (a) Temperature dependence of field-cooled (FC) and zero-field-cooled (ZFC) magnetizations in isotropic Heisenberg systems in various applied fields, for $N=10^3$. M_z is measured relative to its maximum value $M_0 \equiv S$. (b) Effect of anisotropy on FC and ZFC temperature-dependent magnetization M. The value of the DM (■) anisotropy constant $D'=0.25$.

In Fig. 2(a) are plotted FC and ZFC magnetizations versus temperature for an isotropic Heisenberg spin-glass with $J_0=0$. The values of H_z are as indicated. The magnetic fields we consider are taken to be fairly large because the finite size of the system ($N=10^3$) leads to a small but nonvanishing magnetization even at $H=0$. We therefore had to consider applied fields which are large enough to make this effect negligible. The smallest values of H_z we can treat correspond to $H_z \approx 0.1$ in units of $\bar{J}$ which is of the order of 1 kG. However, as discussed in I, reasonable agreement between theory and experiment can only be obtained when the theoretical fields are rescaled by about a factor of 10. While this is consistently found in the Ising case,[22] we speculate that it is also true for vector spins. Thus a more appropriate correspondence is $0.1\bar{J} \approx 100$ G.

All our results are consistent with the observation that the FC and ZFC magnetizations are identical at all H. Magnetic hysteresis calculations, presented in Sec. V, help to reinforce this conclusion. We have verified that the FC and ZFC states are microscopically distinct; however, the

$\vec{m}_i$ are always related by an arbitrary rotation about the z axis. The FC state is found to be microscopically reversible upon subsequent warming, so that, as in the Ising case, there is no minima hopping in the field-cooled process. In the isotropic case, the same result applies to the ZFC state, although we end up in a different rotation of this state, depending upon how we apply the magnetic field after cooling in zero field. This reflects the fact that any application of H below T_c leads to minima hopping. Since there are an infinite number of rotated FC states, when we cool slowly, we always find a different microscopic ground state. These correspond to different initial guesses of the $\vec{m}_i$ at the highest T we use to start our cooling procedure. It should be noted that extremely rapid cooling (quenching) can lead to states other than those belonging to the FC manifold.

The absence of macroscopic irreversibility that we see in the temperature-dependent magnetizations (and in the field-dependent measurements discussed in Sec. V) should be compared with inferences made on the basis of replica-symmetry-breaking models. This approach was first applied to the isotropic infinite-range Heisenberg model by Gabay and Toulouse.[1] Their work was subsequently corrected by Cragg, Sherrington, and Gabay.[2] These authors argued that replica symmetry breaking, which is claimed to be the theoretical indicator of magnetic irreversibility, coincides with the onset of transverse spin-glass order. By random searches at $T=0$, or rapid-quenching procedures, we have found states which do not belong to the manifold of rotated FC states. They are, in agreement with Bray and Moore's[15] conclusions, surprisingly similar to the FC state. Presumably, their temperature onset roughly coincides with that of $Q_\perp(T,H)$. However, the accessibility of the (lower-energy) FC states seems to mask the presence of these other minima, so that they do not lead to irreversible phenomena. In view of these observations it may be that replica symmetry breaking (or the onset of many metastable states) is not as intimately connected with irreversible processes as has been claimed.[1,2]

There are two other explanations of the possible discrepancy between our results and those of Refs. 1 and 2. It may be that the range of the interaction plays an important role in determining the characteristics of irreversible processes. We have carried out mean-field studies of small ($N \lesssim 400$) "infinite"-range interaction spin-glasses.[23] These do appear to have some macroscopic irreversibility. This result may derive from the fact that the barriers between minima become larger as the range of the interaction increases. Thus the FC states are not as accessible as in the finite-range case. Alternatively, it may be that even in the finite-range case when $N \sim 1000$, our systems are not sufficiently large to be representative of the thermodynamic limit. It should be noted, however, that Bray and Moore[15] have found that by $N \sim 1000$ there are several hundred inequivalent ground states for the infinite-range model and even more for the case of finite-range interactions. Finally, we do not believe the use of mean-field theory is a significant factor here, since the absence of magnetic hysteresis (discussed in Sec. V) is observed at $T \approx 0$, in which limit the reaction correction to mean-field theory is negligible.

Despite these possible reservations, it seems clear that irreversibility must be considerably weaker, the more isotropic the spin-glass. Since there are no truly isotropic spin-glasses in nature, the only test of our results is to search for systematic trends in classes of compounds which show varying degrees of anisotropy.[16]

Once anisotropy is introduced we find results for M^{FC} and M^{ZFC} which are qualitatively similar to those found in the Ising model (paper I). As shown in Fig. 2(b), the FC and ZFC magnetizations are split at roughly the temperature at which M^{ZFC} has a maximum. The results shown in the figure are for fairly large $H=0.5$. At this large value of H the ZFC maximum is more evident the larger the anisotropy. The top three sets of figures correspond to the uniaxially anisotropic case, the bottom one is for the case of DM anisotropy with $D' \sim 0.25$. This is also semiquantitatively similar to the uniaxial case for $D \sim 0.25$ It should be seen that the effect of $J_0 \neq 0$ is to raise the magnitude of the magnetization (note the breaks in the vertical scale) and to slightly flatten out the ZFC curve. J_0 acts to enhance the magnetic field, as expected. Comparing Figs. 1(a) and 2(b) with $J_0=0$, $D=0.8$, it may be seen that $Q_\perp$ becomes nonzero at around $T \sim 2.0$. However, the splitting of the FC and ZFC curves occurs at around $T \sim 3.0$. Thus the onset of irreversibility is not associated with the onset of $Q_\perp$. This is not unexpected, since an Ising spin-glass, which has $Q_\perp \equiv 0$, has irreversibility. Obviously this does not contradict the results of Refs. 1 and 2 which deal only with an isotropic spin-glass.

An interesting case of a uniaxially anisotropic system is the limit $D \rightarrow -\infty$, in which the spins are confined to the x-y plane. The results for the FC and ZFC magnetizations are shown in Fig. 3 for $N=10^3$ as a function of temperature. Here the field is applied in the x direction. We see by comparison with the Ising and isotropic Heisenberg models, at this value of $|\vec{H}|$, that the behavior of the ZFC and FC curves is somewhere between that of these two other cases. The applied magnetic field breaks the rotational symmetry in the x-y plane; hence the x-y spin-glass is not expected to have full reversibility, as in the isotropic Heisenberg model. However, because of the extra rotational degrees of freedom of the individual spins, it is reasonable that the irreversibility in the x-y case is not as extreme as for Ising spin-glasses.

V. RESULTS FOR THE MAGNETIC HYSTERESIS CURVES

In the Ising case of paper I we saw that magnetic hysteresis occurs because small changes in H destroy minima on the spin-glass free-energy surface; the system must hop to a nearby state (as in a first-order transition). The same processes occur in an isotropic Heisenberg spin-glass. However, these lead to microscopic, but not macroscopic, irreversibility.

In Fig. 4 are plotted the calculated hysteresis curves for an isotropic Heisenberg system of 10^3 spins. Recall that an "infinitesimal" field is also applied in the $\hat{x}+\hat{y}$ direction, as described above. Each curve corresponds to a different value of J_0. For all values of J_0, including the ferromagnetic case ($J_0=1.0$), the M_z vs H_z curve passes through the origin. There is no macroscopic hysteresis. The temperature T is taken to be 1.0; however, we have verified that the results are qualitatively the same at lower temperatures. Thus the observed absence of hysteresis

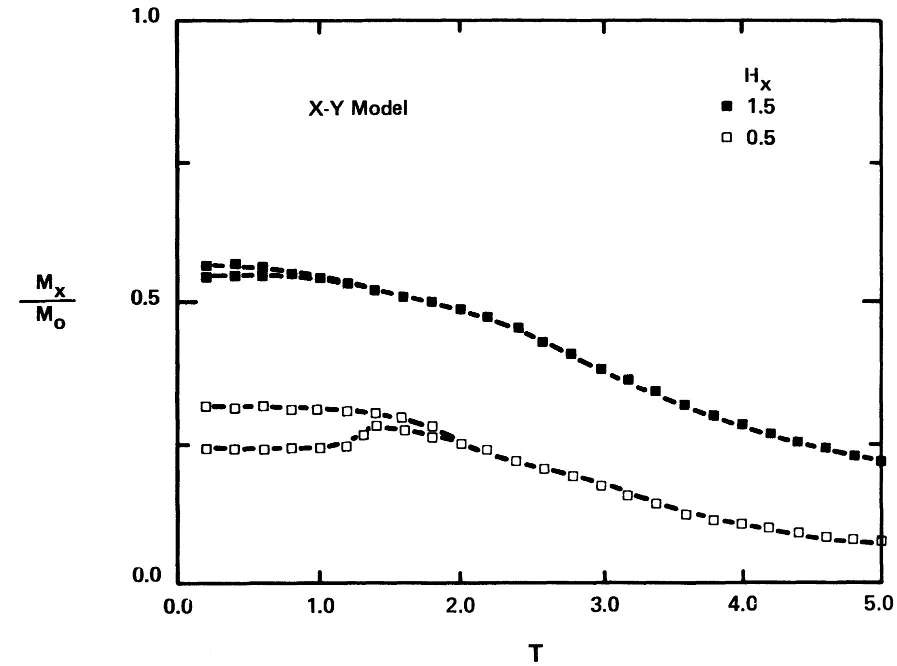


FIG. 3. Temperature dependence of ZFC (bottom curve in each pair) and FC (top curve in each pair) magnetization for x-y model in various fields $\vec{H} \| \hat{x}$.

loops is not affected in any qualitative way by the neglect of the reaction terms (which assumption is valid at $T=0$). When we sweep from negative to positive H we find that at each value of the field the $\vec{m}_i$ are different than was found on the sweep from positive to negative H; however, the states are simply related by a rotation in the x-y plane so that the macroscopic magnetization is unaffected by the history. Furthermore, in a hysteresis calculation each $M_z(H)$ is found to be identical to that obtained by field cooling. This reinforces the observation made in Sec. IV,

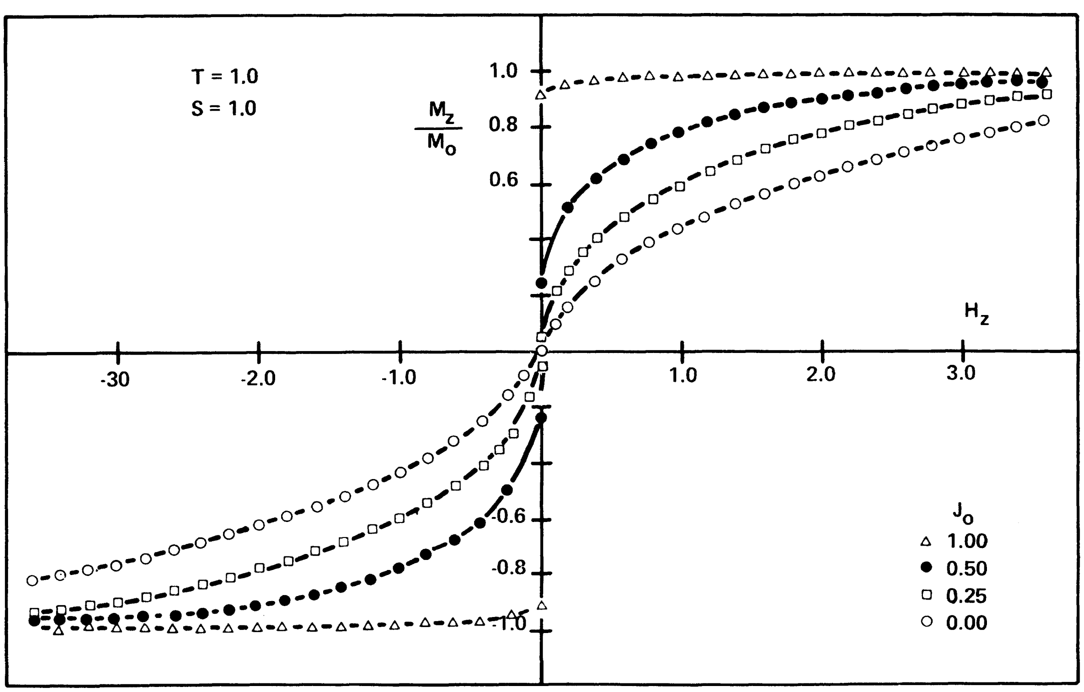


FIG. 4. Magnetization vs field for various J_0 in isotropic Heisenberg system with $N=10^3$. Note $J_0=1.0$ is ferromagnetic.

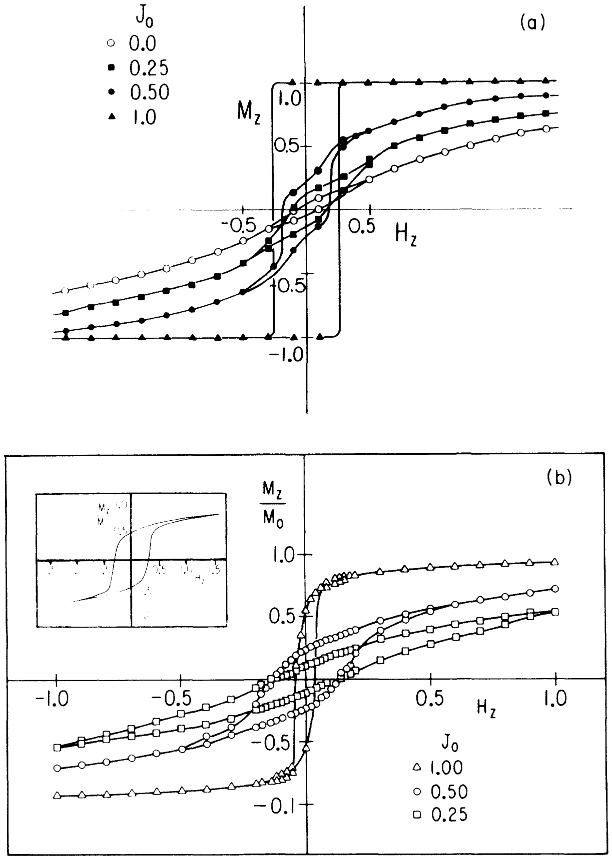


FIG. 5. (a) Magnetic hysteresis curves for case of uniaxial anisotropy with $D=0.4$ and various J_0. Here $N=6^3$. (b) Magnetic hysteresis curves for case of DM anisotropy for $D'=0.5$ for various J_0 and $N=10^3$. Inset shows the effect of a type of combined uniaxial DM anisotropy with J_0 also present (see text).

that because of the rotational degeneracy of the FC state, it is extremely accessible; as a consequence, the spin-glass exhibits no macroscopic irreversibility.

To get additional insight into the behavior of magnetic hysteresis, we have performed hysteresis calculations for *nondisordered* Ising and Heisenberg ferromagnets in mean-field theory. In the Ising case the loop is rectangular, much like that shown in Fig. 5(a) for $J_0=1.0$. This loop corresponds to the fact that there are at most two minima on the free-energy surface. In this case the system stays in the ↑ minimum, until it becomes unstable with decreasing H. It then jumps discontinuously to the ↓ state. There is a small range of H over which two minima coexist, so that magnetic hysteresis represents a supercooling, nonequilibrium phenomenon. In the nondisordered isotropic Heisenberg ferromagnet, the magnetization always points along the external field direction (which is the only direction of broken symmetry). As H_z is turned off, there can be no component of $\vec{M}$ along the z direction so that M_z vanishes when $H_z=0$.[24] The hysteresis behavior observed for spin-glasses seems to be a direct reflection of these results for nondisordered ferromagnets: The Ising system exhibits macroscopic hysteresis, whereas the Heisenberg case does not. Physically, this corresponds to the fact that vector spins have additional flexibility and can "follow" the motion of the external field. They do not get readily trapped into metastable states, as in the Ising case. Thus there is no remanence or macroscopic irreversibility.

In Fig. 5(a) are shown the effects of uniaxial anisotropy on the hysteresis loops for a spin-glass of $N=6^3$ spins. Here the anisotropy constant was fixed at the value $D=0.4$ and the loops are plotted for various J_0. The case $J_0=0$ may be contrasted with the results shown by open circles in Fig. 4. The introduction of uniaxial anisotropy opens up the hysteresis loop, in the same way that it splits the FC and ZFC magnetization as a function of temperature (see Sec. IV).

The effect of increasing J_0 is to make the loops more rectangular. By $J_0=1.0$, the spin-glass is ferromagnetically ordered. As noted above, the loop shown in Fig. 5(a) for this case is characteristic of an Ising ferromagnet; in general, the change in shape of the loops with increasing J_0 is similar to that found in paper I.

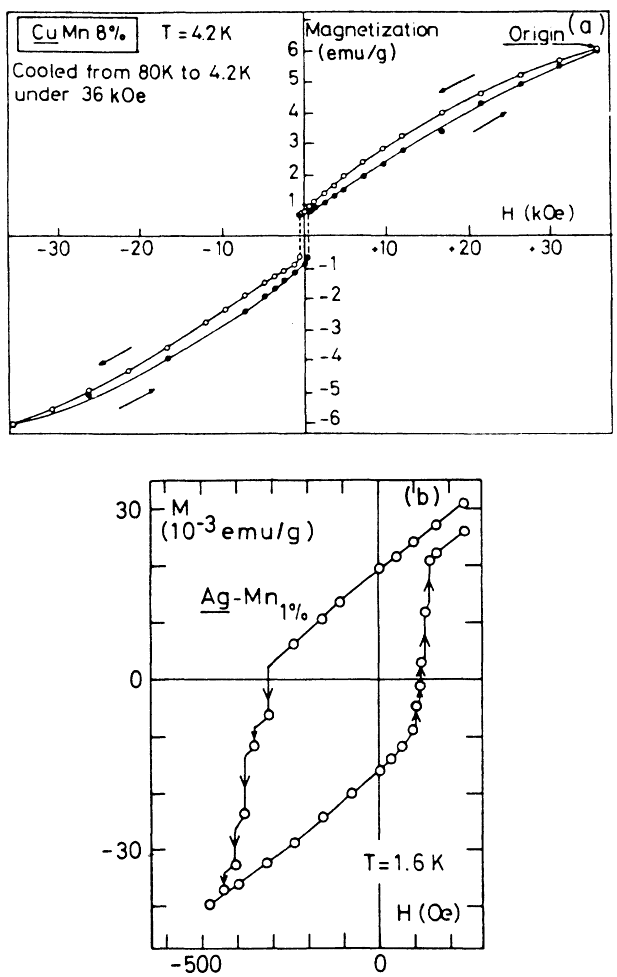


FIG. 6. (a) Measured magnetic hysteresis curve for 8 at. % *Cu*Mn for large field sweep (after Ref. 18). (b) Measured displaced magnetic hysteresis loop in 1 at. % *Ag*Mn (after Ref. 18).

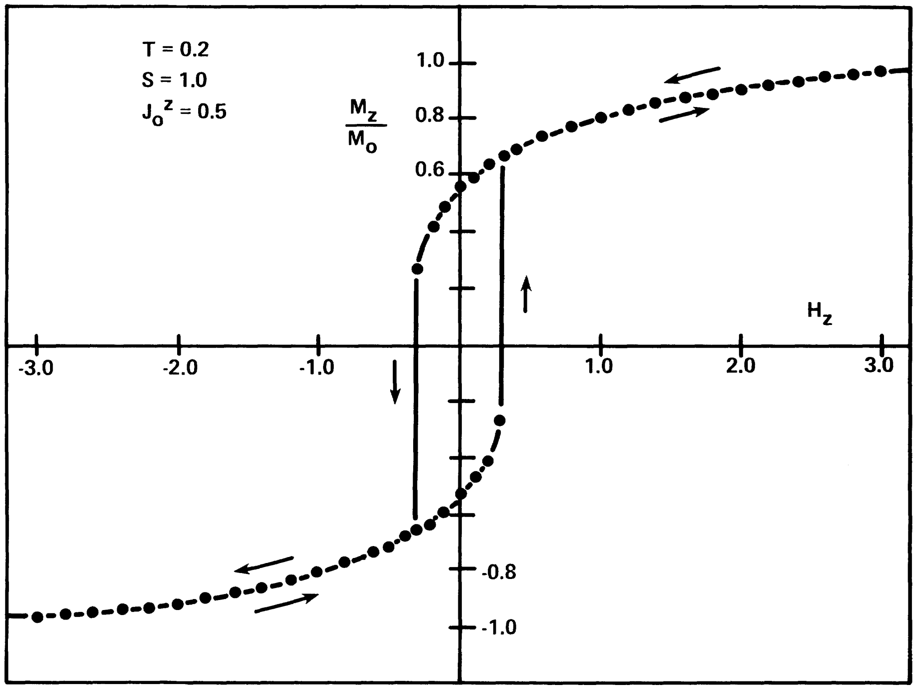


FIG. 7. Magnetic hysteresis loop with a uniaxial anisotropy J_0^z included through the $\{Jij\}$. Note loop tends to have steep reversals as in *Cu*Mn. [See Fig. 6(a)].

When the same type of calculations are repeated in the presence of DM anisotropy, similar results are obtained. Hysteresis loops for this case are shown in Fig. 5(b) in a $10\times10\times10$ spin-glass with anisotropy constant $D'=0.5$ and temperature $T=0.2$. The values of J_0 are 0.25, 0.50, and 1.00. Only the last case has a nonzero spontaneous magnetization. Comparing Figs. 5(a) and 5(b), it may be seen that in the case of DM anisotropy the loops appear to be narrower and less rectangular, particularly at large J_0. This last effect is not unexpected since anisotropy of the DM type $[\vec{D}'_{ij}\cdot\vec{S}_i\times\vec{S}_j]$ will oppose the tendency for cooperative spin reversal in magnetic hysteresis. Hence the abrupt magnetization reversals seen in the uniaxial case are not as readily obtained. A simultaneous treatment of both uniaxial and DM anisotropy is somewhat difficult, although feasible. To mimic a combined uniaxial DM anisotropy which also includes J_0 effects, we have added a uniaxial component to $\overleftrightarrow{J}_{ij}$ in Eq. (2.3), called J_0^z. The resulting hysteresis curve for $J_0^z=0.5$ and all other parameters the same as in Fig. 5(b) is shown in the inset of Fig. 5(b). With the "uniaxial" as well as DM anisotropy present, the loop is broadened and the magnetization reversals are sharper than shown in the main portion of the figure.

It was noted in I that sharp magnetization reversals are seen[17,18] in Mn-containing spin-glasses and in concentrated *Au*Fe alloys.[25] All our calculations based on the anisotropic Heisenberg system (of which the Ising model is a special case) suggest that these sharp reversals are only present when J_0 is positive. Only under these circumstances do the spins flip cooperatively as H is changed. It is clear that such a ferromagnetic tendency is present in *Au*Fe alloys. This was discussed in detail in paper I.

As discussed previously,[3] there is experimental evidence which suggests that Mn-containing alloys also have a ferromagnetic tendency. It is important to note that what determines the sharpness of the hysteresis loop is not the magnitude of J_0 alone; it also depends on how much and what kind of anisotropy there is to compete with the cooperative effects induced by the J_0 term. Thus a system with a large degree of DM anisotropy [as, for example, *Au*Fe (Ref. 7)] may require a larger amount of J_0 to show sharp hysteresis loops. It may be speculated that since *Cu*Mn is considerably less anisotropic,[7] less of a ferromagnetic tendency is needed to cause the abrupt magnetization reversals seen in hysteresis loops.

The experimentally observed hysteresis loops for Mn-containing alloys are illustrated in Fig. 6. Figure 6(a), which shows the results of a symmetric field sweep, makes it clear that in *Cu*Mn the loop is extremely narrow and the field reversal quite dramatic. The closest we have come to reproducing this type of behavior is to add a "uniaxial" J_{ij} component to the Heisenberg Hamiltonian. The resulting loop is shown in Fig. 7 for $J_0^z=0.5$, $T=0.2$, and $N=10^3$.

Of equal, if not greater, interest from an experimental viewpoint, is the presence of the so-called "displaced loops."[17,18] An example of one is shown in Fig. 6(b); although the data[18] is for *Ag*Mn, the behavior is typical of *Cu*Mn spin-glasses as well. These loops are seen when the field sweep is very nonsymmetric or when the spin-glass is field cooled and the loop is generated by reversing the field at sufficiently small (negative) $|\vec{H}|$ values. A "memory" effect leads to a displacement of the loop relative to the $H=0$ axis. To search for this behavior we considered nonsymmetric field sweeps, starting from large positive $\vec{H}$ values and reversing at small (in magnitude) negative $\vec{H}$. For an Ising spin-glass, as pointed out in I,

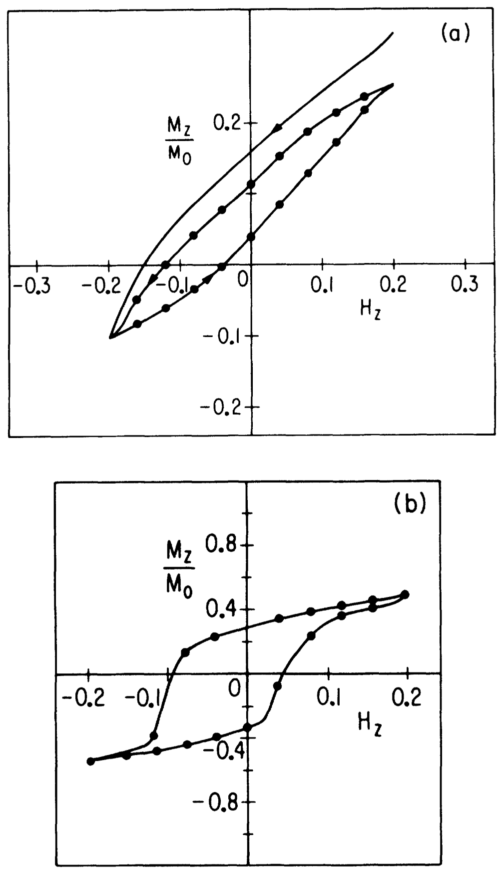


FIG. 8. (a) Magnetic hysteresis loop with DM anisotropy constant $D'=0.5$ for nonsymmetric field sweep with $J_0=0$. (b) Magnetic hysteresis loop with DM anisotropy constant $D'=0.5$ for nonsymmetric field sweep with $J_0=0.75$. This yields a characteristic displaced loop to be compared with Fig. 6(b).

the loop so obtained is primarily shifted upward. When $J_0>0$, the loop is sharp and the asymmetry is only with respect to the $M=0$ axis. We have verified that in the Heisenberg case with uniaxial anisotropy the same qualitative behavior is observed as for Ising spins.

The introduction of DM anisotropy leads to different behavior, however. In Fig. 8(a) is plotted a hysteresis loop with DM anisotropy constant $D'=0.5$, $J_0=0.5$, and $T=0.2$ in an 8^3 Heisenberg spin-glass.[26] The loop is obtained by starting at very large H and then reversing the field at around $-H=0.2$. Only for the case of DM anisotropy have we found that M became positive (as H is increased from negative values) *before* H changed sign to a positive value. In the uniaxial case (which is very much like Fig. 8 in paper I) M changes sign after H has become positive. Only in the DM case is there a kind of "memory effect" which directs the system toward positive magnetization. This can be seen more clearly in Fig. 8(b) for $J_0=0.75$, which is large enough to make the loop more rectangular. Unfortunately, this value of J_0 is sufficient to cause spontaneous ferromagnetism. Here all other parameters are the same as in Fig. 8(a). For more rectangular hysteresis loops, the effect of DM anisotropy is to displace the loop relative to the $H=0$ axis, in a way which is similar to that observed experimentally. By comparing Figs. 6(b) and 8(b) it may be seen that results for displaced hysteresis loops are in reasonable qualitative agreement.

Fert and Levy[7] have argued on the basis of analytical arguments that a unidirectional anisotropy (such as DM) will cause displaced loops. Essential to their derivation is the assumption that the spins all rotate rigidly in response to a rotation of an applied magnetic field. This assumption of rigid rotation can be roughly justified within our framework only when J_0 is sufficiently large. This will be discussed in more detail in the following section. When the spins do not rotate rigidly (i.e., for smaller J_0), then the nature of the "displaced loops" is still characteristically different from that obtained in the uniaxial anisotropy case. Such nonrigid rotations lead to loops like that shown in Fig. 8(a).

In summary, our numerical results demonstrate that under the assumption of rigid spin rotation (which we find will occur when a ferromagnetic tendency is present) the effect of Dzyaloshinsky-Moriya ansiotropy is to yield the so-called displaced hysteresis loops. While it is somewhat disturbing that we need such large values of J_0 to obtain rigid rotations in the DM model, it should be noted that our numerical parameters cannot be quantitatively compared with their experimental counterparts. This is due in part to all the simplifications inherent in our model Hamiltonian, as well as to our (mean-field) approximate solution of this model applied to a finite system.

VI. EFFECTS OF FIELD ROTATIONS

An important assumption in recent theoretical work[10,11] on dynamical properties of spin-glasses is that under the proper circumstances all the spins in the system rotate rigidly in response to a rotation of the external field. This is believed to occur when the spin-glass is cooled in a sufficiently large field so that the remanence is well established and does not change in magnitude but only in direction. Furthermore, it is supposed that the anisotropy is relatively weak.

While our own calculations cannot make contact with dynamical measurements (as in torque[27] or ESR experiments[12]), we can study the validity of the assumption that the spins rotate rigidly.[28] We do this for both the isotropic Heisenberg model and for the case of DM anisotropy. In both cases the inclusion of a J_0 term is found to be important. Our parameters were not chosen to match the experimental configurations, but rather to probe in a qualitative way the changes in the spin-glass states, as magnetic fields are rotated. To test the hypothesis of rigid rotation we chose the cooling field to be in the z direction. The field was then rapidly rotated to be along the x axis. In the case of the isotropic Heisenberg model, the final and initial states are necessarily macroscopically equivalent; one particular microscopic final state can be obtained by rotating all of the original spins by 90° about the y axis. The final state that we found, however, was not this special rotated state. This is not surprising because we allowed only one spin to change at a time in executing our iterative scheme. The final spin-glass state was not ob-

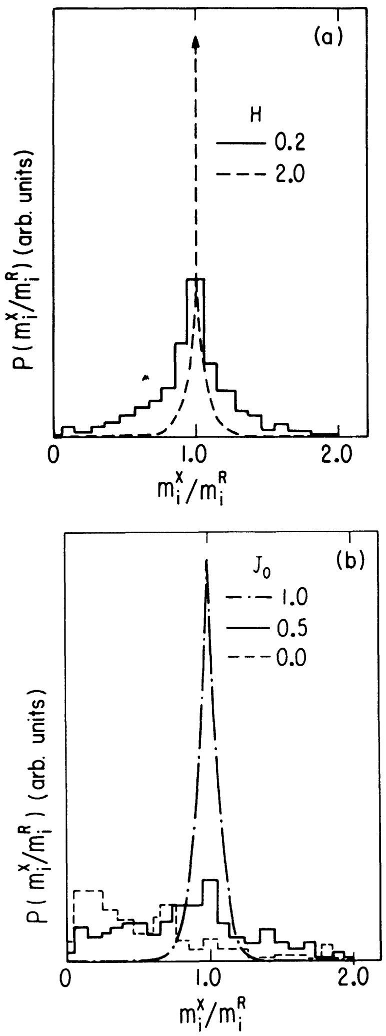


FIG. 9. (a) Distribution of calculated x component of thermally averaged spins $\{\vec{m}_i\}$, divided by value (m_i^R), obtained if m_i were to rotate rigidly in response to H rotation. The field applied to isotropic Heisenberg system (with H values as indicated) is quickly rotated from $\hat{z}$ to $\hat{x}$ direction. A rigid rotation of all spins corresponds to a δ function at 1.0. (b) Distribution of the calculated x component of thermally averaged spins $\{\vec{m}_i\}$ divided by value obtained if $\vec{m}_i$ were to rotate rigidly in response to $\vec{H}$ rotation [same configuration as in (a)].

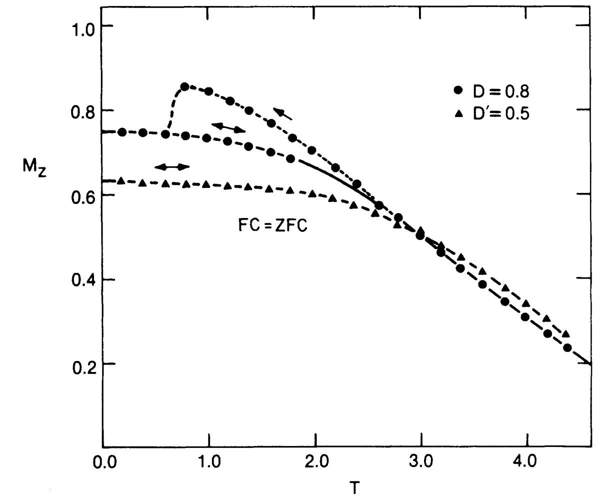


FIG. 10. Temperature dependence of FC and ZFC magnetizations in coexistent ferromagnetic spin-glass. Top curves are for uniaxial anisotropy and bottom for DM case. Here $H=0.5$ and $J_0=0.6$. Dashed curve is most likely a numerical artifact.

tained by a $\pi/2$ rotation (about the y axis) of the initial one, although it was macroscopically equivalent to such a state. The difference arose because the spins also rotated about the x axis by an arbitrary amount which presumably depended on the order of sequencing of the iterations. Consecutive single-spin-flip processes do not necessarily find that path between the initial and final states in which all spins rotate rigidly, although such a path does not involve hopping over barriers. It is not clear what happens in laboratory spin-glasses; it seems plausible that in these highly disorganized systems coherent rotation of all the spins will not generally occur, but rather that individual spins will, in general, flip in an uncoordinated fashion.

To quantify the degree of rigidity of spin rotation in response to field changes, we plotted a histogram of the ratio of the final $\vec{m}_i$ at a given site to the value it would have if it rotated in the same way as the total magnetization. Figure 9(a) illustrates what happens for the isotropic Heisenberg system. The horizontal axis plots the value of m_i^x after rotation, divided by the quantity

$$[(m_i^z)_0\sin\theta+(m_i^x)_0\cos\theta]\equiv m_i^R\ .$$

Here θ is taken to be the angle through which the total magnetization rotates. This latter expression is the expected value of m_i^x if this spin rotated about the y axis by θ. Here the subscript zero refers to the initial values of the spin, before rotation of the field. As expected, we found that for the Heisenberg case $\theta=\pi/2$. The resulting histogram is shown in Fig. 9(a) for two values of $|\vec{H}|=0.2$ and 2.0. While it is peaked around the value 1.0, there is a clear distribution in the histogram, so that all spins have not rotated rigidly. As the field value is increased from 0.2 to 2.0, the distribution sharpens up. This effect is also seen as J_0 is increased. Large fields or positive J_0 tend to keep all the spins aligned so that they rotate rigidly.

The effect of J_0 is even more striking when DM anisotropy is present. In Fig. 9(b) we have plotted the same histogram as in Fig. 9(a) with $D'=0.25$. Here the angle θ is less than $\pi/2$ as expected, since the total magnetization cannot rotate freely to follow the direction of the field. The presence of microscopic anisotropy leads to an effective "anisotropy field"[29] which acts to "resist" rotations of the magnetization. In Fig. 9(b) it is clear that for $J_0=0$ and fairly large D', there is no peak in the distribution at the value 1.0, as would be seen if the spins rotated rigidly. As J_0 increases to 0.5 a small maximum is present. By the time $J_0=1.0$ (which is in the ferromagnetic limit) the spins are clearly rotating rigidly as the field rotates and the distribution is highly peaked. In summary, the assumption of rigid rotation of the spins seems to be justified under those same circumstances which lead to sharp reversals in magnetic hysteresis. (That is, there must be a tendency towards ferromagnetism.) As seen in Sec. V, our results for magnetic hysteresis seem to be consistent with analytical calculations[7] for hysteresis behavior (which are based on rigid-spin rotations). Whenever we also can justify the assumption of rigid-spin rotation, we see displaced loops having the character of those predicted on the basis of analytical arguments.

VII. FERROMAGNETIC SPIN-GLASSES

Gabay and Toulouse[1] (GT) have proposed that transverse spin-glass and longitudinal ferromagnetic order can coexist. In our calculations, we see clear evidence for this coexistence when J_0 is reasonably large ($J_0 \gtrsim 0.5$) and when the uniaxial anisotropy and magnetic fields are sufficiently small that they do not suppress $Q_\perp$ altogether. The evidence regarding the experimental realization of the GT state is not definitive. Mössbauer experiments on *Au*Fe (which probe short-range order) are cited as support for coexistence.[30] Finite-field magnetization data have been interpreted as suggestive of "reentrant ferromagnetism": There is a transition from long-range ferromagnetism (which may or may not be of the coexistent GT type) to spin-glass order as the temperature is lowered.[31–34] Magnetic hysteresis data[35] in these systems also show unusual effects. It has been argued[33] that the reentrant state does not correspond to long-range ferromagnetic order when there is no applied field. Presumably, sufficiently large applied fields will also destroy the low-temperature spin-glass state. Therefore, a truly reentrant state may only exist in a narrow range of fields $H>0$.

Although we cannot address these experimental findings directly, we can determine the theoretical behavior of the FC and ZFC magnetizations for moderate J_0 and finite H, and the zero-field (J_0,T) phase diagram. The temperature-dependent magnetizations are plotted in Fig. 10 for a coexistent ferromagnetic spin-glass in a field of $H=0.5$. Both uniaxial and DM anisotropy are considered and $J_0=0.6$ in both cases; the top pair of curves corresponds to the former and the bottom curve to the latter. For the uniaxial case we believe the result shown by the dashed line is a numerical artifact, deriving from the fact that upon cooling (at constant field) the system remains in a "supercooled" longitudinally ordered state, due to numerical problems.[36] This happens because the self-consistent equations are always satisfied when $m_i^x = m_i^y \equiv 0$. Eventually, at sufficiently low T, transverse order appears discontinuously as shown in Fig. 10 at $T \sim 0.8\bar{J}$. If this cooled state is then heated up, $Q_\perp$ persists up to higher temperatures ($T_{c\perp} \sim 2.2$); for a fixed T, the warming curve has the lower free energy. Furthermore, this same (warming) curve is obtained by a ZFC procedure. While we believe the bottom curve in the pair (for $D=0.8$) is the physical result for the FC magnetization in the uniaxial case, we have presented the "supercooled" curve to illustrate the difficulties that may be encountered in numerical calculations. While it is highly unlikely, it is not impossible that a laboratory spin-glass with strong uniaxial anisotropy may also exhibit these supercooling instabilities.

For the case of DM anisotropy, $T_{c\perp}$ and $T_{c\parallel}$ are virtually identical, and we consequently had no numerical difficulties with supercooling. The bottom curve in Fig. 10 shows that the FC and ZFC curves are indistinguishable for this value of H. In general, because J_0 acts to enhance the field, the temperature-dependent magnetizations are similar to those observed at high fields in "pure" spin-glasses. Hence $M^{\rm ZFC} = M^{\rm FC}$, even for relatively low fields, in the GT state. Because of finite-size effects we were unable to consider fields sufficiently small to see the splitting, if any, of the two history-dependent magnetizations. Experimentally $M^{\rm FC}$ and $M^{\rm ZFC}$ are observed to split in sufficiently low fields. Our calculated hysteresis loops for the GT state are rather boxlike, as in Fig. 5(a) for $J_0=1.0$. We saw no particular effects that could qualitatively distinguish between these loops and those we found for a disordered ferromagnet with no spin-glass order ($Q_\perp \equiv 0$).

That we saw no decrease in the FC magnetization at low T for any J_0 would appear to be inconsistent with experiment. However, because the fields we applied were of sizable magnitude, it may be that they suppressed the low-T spin-glass state. To further test for reentrancy, we studied the phase diagram at zero H in the (J_0,T) plane. The two $T=0$ intercepts corresponding to the spin-glass to GT state and the GT to ferromagnetic state transition can be reliably calculated using our mean-field technique.[37] Presumably the slope of the lines at finite but small T can also be accurately determined. Our results for a 20^3 isotropic Heisenberg spin-glass gave no evidence for reentrancy. That is, the slope of the spin-glass to GT state transition was not found to be negative at low T.

We conclude that in zero and in moderate applied fields the so-called reentrant state is not found in our calculations, although the GT state is clearly in evidence. We cannot rule out the possibility that a reentrant transition exists only in some narrow range of magnetic fields. Alternatively, it may be that the reentrant phenomena observed in laboratory spin-glasses may be due to imhomogeneity, time-dependent, or other effects which are not included in our theoretical model.

VIII. CONCLUSIONS

One of the most striking conclusions presented in this work is that an isotropic (short-range) Heisenberg spin-glass has no macroscopic irreversibility. This is a consequence of the ready accessibility, due to rotational symmetry, of the field-cooled state. Once microscopic anisotropy is introduced, most history-dependent properties are found to be similar to those we presented for the Ising case in the preceding paper.

The new effects in anisotropic Heisenberg spin-glasses (not seen in the Ising case) are (i) the presence of displaced loops (when the anisotropy corresponds to the Dzyaloshinsky-Moriya mechanism) and (ii) the coexistence of (transverse) spin-glass and (longitudinal) ferromagnetic order. We have discussed both of these in some detail. In order to obtain displaced loops which are qualitatively similar to those observed in *Cu*Mn, we found it necessary to introduce a positive J_0. This ensures that the spins rotate rigidly in response to a rotation of the magnetic field. While the assumption of rigid rotation seems to be necessary in order to make progress theoretically[10,11] and to be justified experimentally,[14,27] it is not clear what is going on microscopically to maintain this rigidity. One possible mechanism is a tendency toward short-range ferromagnetic order. When this ferromagnetic tendency is sufficiently strong, we find a Gabay-Toulouse spin-glass ferromagnetic state is obtained from our numerical calculations. However, for zero and moderately high fields, we do not see any evidence for the so-called reentrant behavior which is frequently observed in the laboratory.[31–34]

We have found that microscopic anisotropy is necessary to obtain remanence, magnetic hysteresis, and other irreversible processes. On the other hand, it seems clear experimentally[18] that these properties are intrinsic to spin-glasses and, for example, do not change appreciably[18] as further anisotropy is introduced through the addition of magnetic impurities. Our point of view is not necessarily inconsistent with these experimental findings. Anisotropy must be present in order to inhibit the vector spin from "following" a magnetic field. No spin-glass can be entirely free from intrinsic anisotropy. The addition of impurities, which is an extrinsic effect, does not seem likely to further enhance irreversibility, once it is reasonably well established.

We have not discussed the concept of macroscopic anisotropy, in part because there is no unique definition of this property. Our approach has focused entirely on microscopic anisotropy mechanisms. While ours is among the first attempts to relate microscopic anisotropy to irreversible processes, it is clear that further studies are needed, in particular, to probe the barrier heights on the free-energy surface. These should help to give useful insights into possible differences between finite- and infinite-range Heisenberg systems and to provide a more microscopic picture of dynamical effects. This latter is extremely important in view of the fascinating spin resonance[12] and torque experiments[27] which have recently been performed.

We conclude with some speculative remarks concerning the differences observed in magnetic hysteresis and dynamical properties in the two prototypical spin-glass alloys: *Au*Fe and *Cu*Mn. Our explanation for the rigid rotations found in Mn-containing alloys is based on the existence of ferromagnetic correlations. As discussed in I, there is experimental evidence in support of these effects. However, Fe-containing alloys presumably have even greater ferromagnetic tendencies than those containing Mn. The relatively weaker rigidity observed in *Au*Fe, as compared with *Cu*Mn, must then derive from the larger DM anisotropy in the former case.[8] We have shown how this anisotropy destroys cooperative spin reversals in magnetic hysteresis, and in general weakens rigidity [see Figs. 5(b) and 9(b)]. We propose that the microscopic parameter responsible for rigidity is the ratio J_0/D', where D' is the amplitude of the Dzyaloshinsky-Moriya anisotropy constant, and J_0 is the positive displacement of the (Gaussian) distribution of exchange interactions.

ACKNOWLEDGMENTS

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[36]In all cases except those discussed in this section, a small perpendicular field was sufficient to eliminate these supercooling problems. Furthermore, with $D=0$, we found a weak DM anisotropy ($D' \sim 0.01$) will suppress the supercooled state; this indicates that it is most likely an artifact.

[37]When the convergence criterion was not sufficiently stringent, we found that at $T=0$ some pure spin-glass ground states could exist in the GT region of the phase diagram. This may be interpreted to suggest that in this region there are "pseudo minima" of F having zero spontaneous magnetic moment. Whether this is relevant to reentrant phenomena is unclear.